

# The interaction region of high energy protons

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## Abstract

The spatial view of the interaction region of colliding high energy protons (in terms of impact parameter) is considered. It is shown that the region of inelastic collisions has a very peculiar shape. It saturates for central collisions at an energy of 7 TeV. We speculate on the further evolution with energy, which is contrasted to the "black disk" picture.

## 1 Introduction

The search ever deeper into the interior of matter successfully started by Rutherford's discovery of atomic structure is going on now at much lower scales (below  $10^{-13}$  cm) at high energy accelerators. The interaction region of colliding protons can be quantitatively explored with the help of the unitarity condition if experimental data on their elastic scattering are used. With only these two ingredients at hand we are able to show that the energy evolution of the inelastic interaction region demonstrates quite surprising features.

## 2 The unitarity condition

From the theoretical side, the most reliable information comes from the unitarity condition. The unitarity of the  $S$ -matrix  $SS^+=1$  relates the amplitude of elastic scattering  $f(s, t)$  to the amplitudes of inelastic processes  $M_n$ . In the  $s$ -channel they are subject to the integral relation (for more details see, e.g., [1, 2, 3]) which can be written symbolically as

$$\text{Im}f(s, t) = I_2(s, t) + g(s, t) = \int d\Phi_2 f f^* + \sum_n \int d\Phi_n M_n M_n^*. \quad (1)$$

The variables  $s$  and  $t$  are the squared energy and transferred momentum of colliding protons in the center of mass system  $s = 4E^2 = 4(p^2 + m^2)$ ,  $-t = 2p^2(1 - \cos \theta)$  at the scattering angle  $\theta$ . The non-linear integral term represents the two-particle intermediate states of the incoming particles. The second term represents the shadowing contribution of inelastic processes to the imaginary part of the elastic scattering amplitude. Following [4] it is called the overlap function. This terminology is ascribed to it because the integral there defines the overlap within the corresponding phase space  $d\Phi_n$  between the matrix element  $M_n$  of the  $n$ -th inelastic channel and its conjugated counterpart with the collision axis of initial particles deflected by an angle  $\theta$  in proton elastic scattering. It is positive at  $\theta = 0$  but can change sign at  $\theta \neq 0$  due to the relative phases of inelastic matrix elements  $M_n$ 's.

At  $t = 0$  it leads to the optical theorem

$$\text{Im}f(s, 0) = \sigma_{tot}/4\sqrt{\pi} \quad (2)$$

and to the general statement that the total cross section is the sum of cross sections of elastic and inelastic processes

$$\sigma_{tot} = \sigma_{el} + \sigma_{in}, \quad (3)$$

i.e., that the total probability of all processes is equal to one.

### 3 The geometry of the interaction region

Here, we show that it is possible to study the space structure of the interaction region of colliding protons using information about their elastic scattering within the unitarity condition. The whole procedure is simplified because in the space representation one gets an algebraic relation between the elastic and inelastic contributions to the unitarity condition in place of the more complicated non-linear integral term  $I_2$  in Eq. (1).

To define the geometry of the collision we must express all characteristics presented by the angle  $\theta$  and the transferred momentum  $t$  in terms of the transverse distance between the trajectories of the centers of the colliding protons - namely the impact parameter,  $b$ . This is easily carried out using the Fourier - Bessel transform of the amplitude  $f$  which retranslates the momentum data to the corresponding transverse space features and is written as

$$i\Gamma(s, b) = \frac{1}{2\sqrt{\pi}} \int_0^\infty d|t| f(s, t) J_0(b\sqrt{|t|}). \quad (4)$$

The unitarity condition in the  $b$ -representation reads

$$G(s, b) = 2\text{Re}\Gamma(s, b) - |\Gamma(s, b)|^2. \quad (5)$$

The left-hand side (the overlap function in the  $b$ -representation) describes the transverse impact-parameter profile of inelastic collisions of protons. It is just the Fourier – Bessel transform of the overlap function  $g$ . It satisfies the inequalities  $0 \leq G(s, b) \leq 1$  and determines how absorptive the interaction region is, depending on the impact parameter (with  $G = 1$  for full absorption and  $G = 0$  for complete transparency). The profile of elastic processes is determined by the subtrahend in Eq. (5). If  $G(s, b)$  is integrated over all impact parameters, it leads to the cross section for inelastic processes. The terms on the right-hand side would produce the total cross section and the elastic cross section, correspondingly, as should be the case according to Eq. (3). The overlap function is often discussed in relation with the opacity (or the eikonal phase)  $\Omega(s, b)$  such that  $G(s, b) = 1 - \exp(-\Omega(s, b))$ . Thus, full absorption corresponds to  $\Omega = \infty$  and complete transparency to  $\Omega = 0$ .

The most prominent feature of elastic scattering is the rapid decrease of the differential cross section with increasing transferred momentum,  $|t|$ , in the diffraction peak. As a first approximation, at present energies, it can be described by the exponential shape with the slope  $B(s)$ :

$$\frac{d\sigma}{dt} = \frac{\sigma_{tot}^2}{16\pi} \exp(-B(s)|t|). \quad (6)$$

The diffraction cone contributes predominantly to the Fourier - Bessel transform of the amplitude. Using the above formulae, one can write the dimensionless  $\Gamma$  as

$$i\Gamma(s, b) = \frac{\sigma_t}{8\pi} \int_0^\infty d|t| \exp(-B|t|/2)(i + \rho) J_0(b\sqrt{|t|}). \quad (7)$$

Here, the diffraction cone approximation (6) is inserted. Herefrom, one calculates

$$\text{Re}\Gamma(s, b) = \zeta \exp(-\frac{b^2}{2B}), \quad (8)$$

where we introduce the dimensionless ratio of the cone slope (or the elastic cross section) to the total cross section

$$\zeta = \frac{\sigma_{tot}}{4\pi B} = \frac{4\sigma_{el}}{(1 + \rho^2)\sigma_{tot}} \approx \frac{4\sigma_{el}}{\sigma_{tot}}. \quad (9)$$

Table. The energy behavior of  $\zeta$  and  $G(s, 0)$ .

$\sqrt{s}$ , GeV	2.70	4.11	4.74	7.62	13.8	62.5	546	1800	7000
$\zeta$	1.56	0.98	0.92	0.75	0.69	0.67	0.83	0.93	1.00-1.02
$G(s, 0)$	0.68	1.00	0.993	0.94	0.904	0.89	0.97	0.995	1.00

The ratio  $\sigma_{el}/\sigma_{tot}$  defines the survival probability of initial protons. The approximation sign refers to the neglected factor  $1+\rho^2$  where  $\rho$  is the ratio of the real to imaginary part of the amplitude in the diffraction cone. In what follows we neglect  $\rho$  according to experimental data (with  $\rho(7 \text{ TeV}, 0) \approx 0.145$ ) and theoretical considerations which favor its decrease inside the diffraction cone. Thus one gets

$$G(s, b) = \zeta \exp(-\frac{b^2}{2B}) [2 - \zeta \exp(-\frac{b^2}{2B})]. \quad (10)$$

The inelastic profile depends on two measured quantities - the diffraction cone width  $B(s)$  and its ratio to the total cross section,  $\zeta$ . It scales as a function of  $b/\sqrt{2B}$ .

For central collisions with  $b = 0$  one gets

$$G(s, b = 0) = \zeta(2 - \zeta). \quad (11)$$

This formula is very significant because it follows herefrom that the darkness at the very center is fully determined by only one parameter,  $\zeta$ , which is the ratio of experimentally measured quantities. It is given by the ratio of the width of the diffraction cone  $B$  (or  $\sigma_{el}$ ) to the total cross section. The energy evolution of these quantities defines the evolution of the absorption value. The interaction region becomes completely absorptive  $G(s, 0) = 1$  in the center only at  $\zeta = 1$  and the absorption diminishes for other values of  $\zeta$ . However for small variations of  $\zeta = 1 \pm \epsilon$  the value of  $G(s, 0) = 1 - \epsilon^2$  varies even less.

In the Table, we show the energy evolution of  $\zeta$  and  $G(s, 0)$  for  $pp$  and  $p\bar{p}$  scattering as calculated from experimental data about the total cross section and the diffraction cone slope at corresponding energies. Let us point out that starting from ISR energies the value of  $\zeta$  increases systematically and at LHC energies becomes equal to 1 within the accuracy of measurements of  $B$  and  $\sigma_{tot}$ .

The impact parameter distribution of  $G(s, b)$  (10) has its maximum at  $b_m^2 = 2B \ln \zeta$  with full absorption  $G(b_m) = 1$ . Its position depends both on  $B$  and  $\zeta$ .

Note, that, for  $\zeta < 1$  (which is the case, e.g., at ISR energies) one gets incomplete absorption  $G(s, b) < 1$  at any physical  $b \geq 0$  with the largest value reached at  $b = 0$  because the maximum appears at non-physical values of  $b$ . The disk is semi-transparent.

At  $\zeta = 1$ , which is reached at 7 TeV, the maximum is positioned exactly at  $b = 0$ , and full absorption occurs there, i.e.  $G(s, 0) = 1$ . The disk center becomes impenetrable (black). The strongly absorptive core of the inelastic interaction region grows in size as we see from expansion of Eq. (10) at small impact parameters:

$$G(s, b) = \zeta[2 - \zeta - \frac{b^2}{B}(1 - \zeta) - \frac{b^4}{4B^2}(2\zeta - 1)]. \quad (12)$$

The term proportional to  $b^2$  vanishes at  $\zeta = 1$ , and  $G(b)$  develops a plateau which extends to quite large values of  $b$  (about 0.5 fm). The plateau is very flat because the last term starts to play a role at 7 TeV (where  $B \approx 20 \text{ GeV}^{-2}$ ) only for larger values of  $b$ .

At  $\zeta > 1$ , the maximum shifts to positive physical impact parameters. A dip is formed at  $b=0$  leading to a concave shaped inelastic interaction region - approaching a toroidal shape. This dip becomes deeper at larger  $\zeta$ . The limiting value  $\zeta = 2$  leads to complete transparency at the center  $b = 0$ .

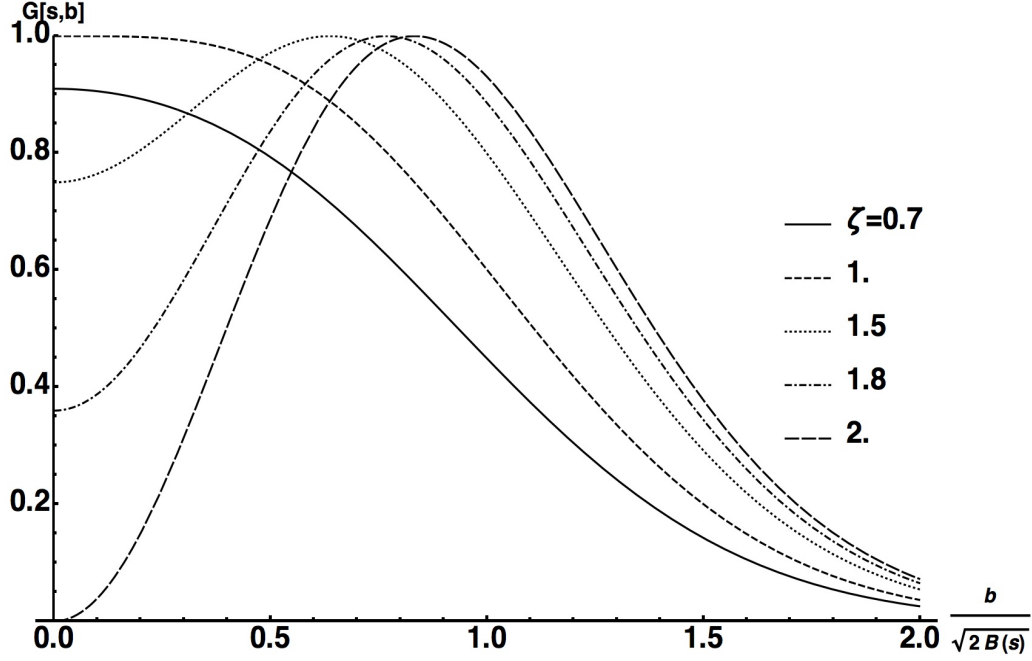
All these cases are demonstrated in Fig. 1 where  $G(s, b)$  is plotted as a function of the scaling variable  $b/\sqrt{2B}$  for different values of the parameter  $\zeta$  according to Eq. (10). The line with  $\zeta = 0.7$  corresponds to ISR results and with  $\zeta = 1$  to LHC. Earlier it was shown that the results of analytical calculations according to (10) and the computation with experimental data directly inserted in the unitarity condition practically coincide (see Fig. 1 in [11]).

What can we expect at higher energies?

The profiles shown in Fig. 1 are valid so long as we can assume that the differential cross section of elastic scattering decreases exponentially with  $|t|$  within the diffraction cone. They can change if this traditional behavior is no longer valid at higher energies. Slope variations of the order of 1 per cent found at 8 TeV by TOTEM [12] are still not significant.

Only guesses can be obtained from the extrapolation of results at lower energies to new regimes, even though experience shows how indefinite and

Figure 1: The evolution of the inelastic interaction region in terms of the survival probability. The values  $\zeta = 0.7$  and  $1.0$  correspond to ISR and LHC energies and agree well with the result of detailed fitting to the elastic scattering data [5, 6, 7]. A further increase of  $\zeta$  leads to the toroid-like shape with a dip at  $b = 0$ . The values  $\zeta = 1.5$  are proposed in [8, 9] and  $\zeta = 1.8$  in [10] as corresponding to asymptotical regimes. The value  $\zeta = 2$  corresponds to the "black disk" regime ( $\sigma_{el} = \sigma_{inel} = 0.5\sigma_{tot}$ ).



even erroneous such extrapolations can be.

First, one may assume that  $\zeta$  will increase without crossing 1 but approaching it asymptotically. That would imply that its precise value at 7 TeV is still slightly lower than 1 within the present experimental errors<sup>1</sup>. Then the inelastic profile shown in Fig. 1 for  $\zeta = 1$  will be quite stable with a slow approach to complete blackness in central collisions and a steady increase of its range. This situation seems most appealing to our theoretical intuition.

However, given the experimentally observed increase of the share of elastic scattering from ISR to LHC, it is tempting to consider another intriguing possibility- that there could be a further increase at still higher energy. Then the interaction region inevitably acquires a toroid-like shape with a dip at the very center ( $b = 0$ ). Some extrapolations of fits at lower energies are presented in [8, 9] and theoretical speculations are discussed in [10]. The line with  $\zeta = 1.5$  describes the profile of the inelastic interaction region according to asymptotic expectations predicted in [8, 9], where successful fits to present experimental data are reported. The new proposal of [10] is shown at  $\zeta = 1.8$ . The dip increases at larger  $\zeta$  and reaches the very bottom  $G(0) = 0$  for  $\zeta = 2$ . Strangely enough this situation with  $\sigma_{el} = \sigma_{inel} = 0.5\sigma_{tot}$  is usually referred to as the "black disk" limit [14].

Protons become impenetrable when  $b = 0$  for  $\zeta = 2$  and only undergo elastic scattering. It is discussed in Ref. [15]. This condition results in purely backward scattering (as in head-on collisions of billiard balls).

In conclusion, we can state that, analyzing the unitarity condition, we have found a special role for the ratio of elastic to total cross sections being equal to 1/4 in 7 TeV pp-interactions and described the consequences of its energy evolution. This role could be attributed to an equal share of processes with exchange and no-exchange of quantum numbers in particle collisions. Then elastic processes constitute a half of the no-exchange share. Another half would be attributed to inelastic diffraction processes. That would lead to saturation of the Pumplin bound [16] which states that their sum is less or equal to 0.5 of the total cross section. However there is still no consensus among experiments about the saturation of the bound at 7 TeV (see [17, 18]). Although there is currently a large latitude for the inelastic diffractive cross sections permitted by the accuracy of the experiments, as was pointed out

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<sup>1</sup> The value of  $\sigma_{el}/\sigma_{tot} = 0.257 \pm 0.005$  reported in [13] would imply  $\zeta = 1.01$  with an uncertainty of  $\sim 2\%$ .

in [19] the value presented in [18] corresponds to perfect agreement with the above picture( ie the above-metioned saturation).

In general, inelastic diffraction is determined by the dispersion of matrix elements while only their averages enter into Eqs (10), (11). Some models have to be invoked in order to predict the dispersion. On a qualitative level it looks as though the absorptive structure of protons is extremely inhomogeneous [20]. That could explain the behavior of the inelastic profile described above.

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## References

- [1] PDG group, China Phys. C **38**, 090513 (2014).
- [2] I.V. Andreev, I.M. Dremin, ZhETF Pis'ma **6** (1967) 810
- [3] I.M. Dremin, Physics-Uspekhi **56** (2013) 3; **58** (2015) 61
- [4] L. Van Hove, Nuovo Cimento **28** (1963) 798
- [5] U. Amaldi, K.R. Schubert, Nucl. Phys. B **166** (1980) 301
- [6] I.M. Dremin, V.A. Nechitailo, Nucl. Phys. A **916** (2013) 241.
- [7] A. Alkin, E. Martynov, O. Kovalenko, S.M. Troshin, Phys. Rev. D **89** (2014) 091501(R)
- [8] A.K. Kohara, E. Ferreira, T. Kodama, pp elastic scattering at LHC energies; arXiv:1408.1599
- [9] D.A. Fagundes, M.J. Menon, P.V.R.G. Silva, Exploring central opacity and asymptotic scenarios in elastic hadron scattering; arXiv:1509.04108
- [10] S.M. Roy, A two component picture for high energy scattering: unitarity, analiticity and LHC data; arXiv:1602.03627
- [11] M.Yu. Azarkin, I.M. Dremin, M. Strikman, Phys. Lett. B **735** (2014) 244; arXiv:1401.1973



- [12] TOTEM Collaboration, Nucl. Phys. B **899** (2015) 527
- [13] TOTEM Collaboration, EPL **101** (2013) 21004
- [14] M.M. Block, F. Halzen, Phys. Rev. Lett. **107** (2011) 212002
- [15] S.M. Troshin, N.E. Tyurin, Phys. Lett. B **316** (1993) 175
- [16] J. Pumplin, Phys. Rev. D **8** (1973) 2899
- [17] CMS Collaboration, Phys. Rev. D **92** (2015) 012003
- [18] ALICE Collaboration, EPJ C **73** (2013) 2456
- [19] P. Lipari, M. Lusignoli, EPJ C **73** (2013) 2630
- [20] K. Fialkowski, H.I. Miettinen, Nucl. Phys. B **103** (1976) 247